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# THESIS

Implementation of a Reliability Shorthand
on the TI-59 Handheld Calculator

by

Hans-Eberhard Peters

October 1982

Thesis Advisor: J.D.Esary

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Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be

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# Implementation of a Reliability Shorthand on the TI-59 Handheld Calculator

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It is shown how a reliability shorthand can be implemented on a handheld calculator.

Assuming constant failure rates, basic structures are used to show how the shorthand can be applied. Several examples are worked out that show, how, with component failure rates as input, a handheld calculator can be used to compute the reliability of a system.

Two TI-59 programs are provided as a computational aid.

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# I. INTRODUCTION

Systems and components can be in either of two states: either they are functioning or they have failed. The ability, that a system stays functioning over a predetermined time interval is called its reliability. It is generally not realistic to assume that a system, say a lightbulb, will fail at a specified time, but rather that T, the time to failure, is a random variable which has a probability distribution that can be specified. The probability distribution for a time to failure is called its life distribution. In this paper we will solely be concerned with one specific type of life distribution which is especially important in reliability theory and practice, the exponential distribution. It has the property that the remaining life of a used component is independent of its age (the "memoryless" property), i.e. a functioning component is always as good as new, the failure rate is constant. The memoryless property is the basis for a reliability shorthand, one that can be implemented on a handheld calculator.

Depending on the size, structure and life distribution of a system, probability statements about its time to

failure are in general not easily achieved. Forming the sum of independent life lengths (i.e. convolving the corresponding life distributions) requires knowledge of integral calculus and computations can become rather tedious.

In the case of the exponential distribution, though, computations can be simplified by translating the problem into a simple shorthand notation and using this shorthand as input for some computing device.

In this paper we will show how a reliability shorthand can be implemented on a handheld calculator. Basic structures are used to show how the shorthand can be applied. Two TI-59 programs are provided as a computational aid. Formulas for the convolution of up to four exponential random variables can be found in Appendix A. Appendix B contains a user guide to the TI-59 programs.

## II. THE CONCEPT OF A RELIABILITY SHORTHAND

### A. BASIC NOTATION

The survival function of a life length can be derived from the distribution function.

Let

$T$ : life length

$F(t) = P(T \leq t)$ be the distribution function of $T$

Then

$$\bar{F}(t) = P(T>t)$$
$$= 1-F(t)$$

is the survival function of T.

In the case of the exponential distribution, $\bar{F}(t) = e^{-\lambda t}$, where $\lambda$ is the failure rate. Translated into shorthand, the life distribution is denoted

$$EXP(\lambda).$$

### B. CONVOLUTION OF DISTRIBUTIONS

When independent random lives are summed up, the corresponding life distributions have to be convolved to determine the probability that the sum of the lives will exceed a specified time t. Let

$T_1, T_2$ : independent life lengths

$\bar{F}_1(t), \bar{F}_2(t)$ : the corresponding survival functions

$f_1(t), f_2(t)$ : the corresponding density functions

$T = T_1 + T_2$ : the total life length

Then

$$\bar{F}(t) = P(T>t)$$
$$= P(T_1 + T_2 > t)$$
$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s) ds.$$

This means that T will exceed a specified time t when

-either $T_1$ exceeds t

-or $T_1$ is smaller than t, say equal to s, and $T_2$ exceeds t-s.

Integration with respect to s (i.e. summing over all possible values of s ) is called the convolution of $T_1$ and $T_2$. When $T_1$ and $T_2$ are both exponentially distributed with failure rates $\lambda_1$ and $\lambda_2$ , i.e.

$$\bar{F}_1(t) = e^{-\lambda_1 t}$$
$$\bar{F}_2(t) = e^{-\lambda_2 t},$$

then the survival function of T is

$$\bar{F}(t) = e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s} \quad ds.$$

Translated into shorthand, the survival function is denoted

EXP ( $\lambda_1$) + EXP ( $\lambda_2$ ).

This shorthand notation is heuristically apparent. We can visualize a 1 component / 1 spare system with EXp($\lambda_1$) and Exp($\lambda_2$) lives respectively. From component 1 the system has an EXP($\lambda_1$) life to begin with. When component 1 fails, the system has an extra EXP($\lambda_2$) life.

C. MIXTURE OF DISTRIBUTIONS

1. MIX-Notation

In the previous chapter, we formed the sum of independent random lives, which each had weight one, i.e.

$$T = T_1 + T_2 .$$

Now consider

$$T = \begin{cases} T_1 & \text{with probability } p_1 \\ T_2 & \text{with probability } p_2 \end{cases}$$

where $p_1 + p_2 = 1$ .

Let $D_1$ and $D_2$ be the probability distributions of the random variables $T_1$ and $T_2$ respectively. The corresponding survival functions are $\overline{F}_1(t)$ and $\overline{F}_2(t)$.

Then

$$\overline{F}(t) = p_1 \overline{F}_1(t) + p_2 \overline{F}_2(t) .$$

In shorthand, the mixture of distributions $D_1$ and $D_2$ with respect to the mixing probabilities $p_1$ and $p_2$ is denoted

$$\text{MIX}[\, p_1 D_1 \;,\; p_2 D_2 \,].$$

## 2. Distributive Law

Now let

$$T = T_3 + T'$$

where

$$T' = \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

Then

$$T = T_3 + \begin{cases} T_1 \text{ with probability } p \\ T_2 \text{ with probability } 1-p. \end{cases}$$

$$T = \begin{cases} T_3 + T_1 \quad \text{with probability } p \\ T_3 + T_2 \quad \text{with probability } 1-p. \end{cases}$$

The distributive law holds due to the fact that the sum of the mixing probabilities for $T_1$ and $T_2$ is equal to one. The survival function of $T$ can be found by convolution:

$$\bar{F}(t) = \bar{F}_3(t) + \int_0^t (p\bar{F}_1(t-s) + (1-p)\bar{F}_2(t-s))f_3(s)ds.$$

With $D_1$, $D_2$, $D_3$ being the probability distributions for $T_1$, $T_2$, $T_3$, the distributive law can be applied to the shorthand notation:

$$D_3 + \text{MIX}\,[pD_1, (1-p)D_2] = \text{MIX}\,[\ p((D_1 + D_3),$$
$$(1-p)(D_2 + D_3)\,].$$

Graphically this can be represented as follows:

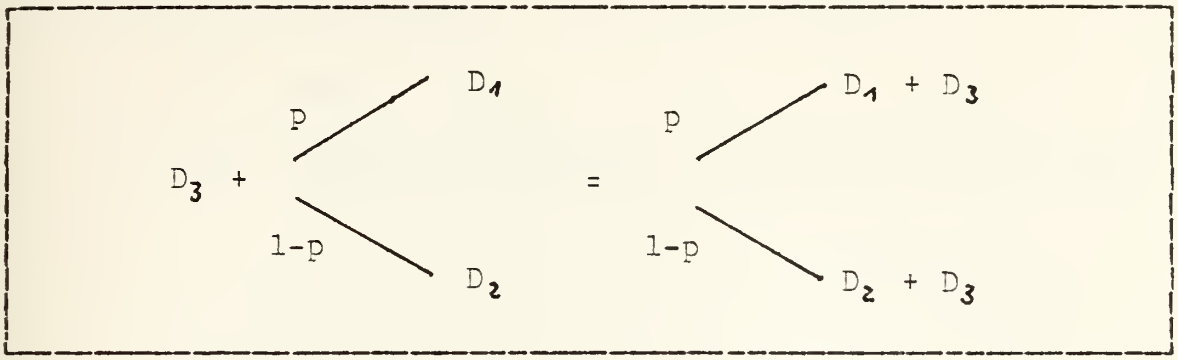



Figure 1: Distributive Property of the MIX-Notation

3. <u>Degeneracy at the Origin</u>

Let

$$P(T=0) = 1.$$

Then the distribution of T is degenerate at zero.

In shorthand notation, such a distribution is called the ZERO-distribution.

Now let $T = T_1 + T_0$

where $T_1$ and $T_0$ have probability distributions $D_1$ and ZERO and survival functions $\bar{F}_1(t)$ and $\bar{F}_0(t)$ respectively.

Then

$$\begin{aligned} \bar{F}(t) &= \bar{F}_1(t) + \int_0^t \bar{F}_0(t-s)\, f_1(s)\, ds \\ &= \bar{F}_1(t). \end{aligned}$$

The ZERO-distribution doesn't add anything to another distribution, so for instance

$$D_1 + ZERO = D_1$$

$$D_2 + MIX[pD_1, (1-p)ZERO] = MIX[p(D_1 + D_2), (1-p)D_2].$$

# III. APPLYING A RELIABILITY SHORTHAND

After this brief survey over the concept of a reliability shorthand we will now show how the shorthand can be applied. To do so we will use basic structures. Part A of this chapter will give examples whose representation in shorthand requires only basic notation described in Chapter II, Parts A and B, whereas Part B of this chapter will give examples whose representation in shorthand makes use of the MIX-notation and the ZERO-distribution.

## A. SUMS OF EXPONENTIALS WITH WEIGHT ONE

### 1. Simple Series System

A series system is a system which is functioning, when all its components are functioning. A two-component series system can be graphically represented as shown in Fig.2 .

Let

$T$ : life of the system

$T_1$: life of component 1

$T_2$: life of component 2

$\bar{F}_1(t)$ = survival function of component 1

$= e^{-\lambda_1 t}$

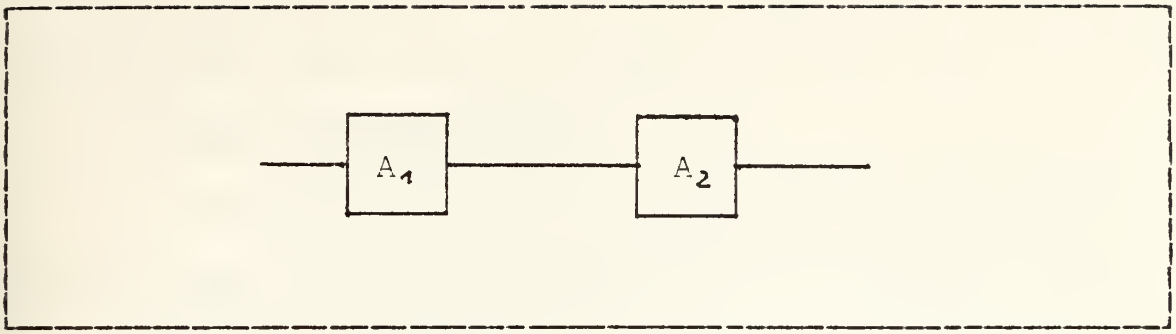



Figure 2: Two-Component Series System

$$\bar{F}_2(t) = \text{survival function of component 2}$$
$$= e^{-\lambda_2 t} .$$

Then

$$T = \min(T_1, T_2)$$

$$\bar{F}(t) = \text{survival function of the system}$$

$$= P(\min(T_1, T_2) > t)$$

$$= P(T_1 > t, T_2 > t)$$

Assuming independence of the two components

$$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t)$$

$$= \bar{F}_1(t)\ \bar{F}_2(t)$$

$$= e^{-\lambda_1 t}\ e^{-\lambda_2 t}$$

$$= e^{-(\lambda_1 + \lambda_2) t} .$$

The shorthand notation for this system is

$$\text{EXP}\ (\lambda_1 + \lambda_2).$$

This is intuitively apparent, as the system has an exponential survival function with failure rate $\lambda_1 + \lambda_2$ .

2. Simple Parallel System

A parallel system is a system which is functioning, when at least one of its components is functioning. A two-component parallel system can be graphically represented as follows:

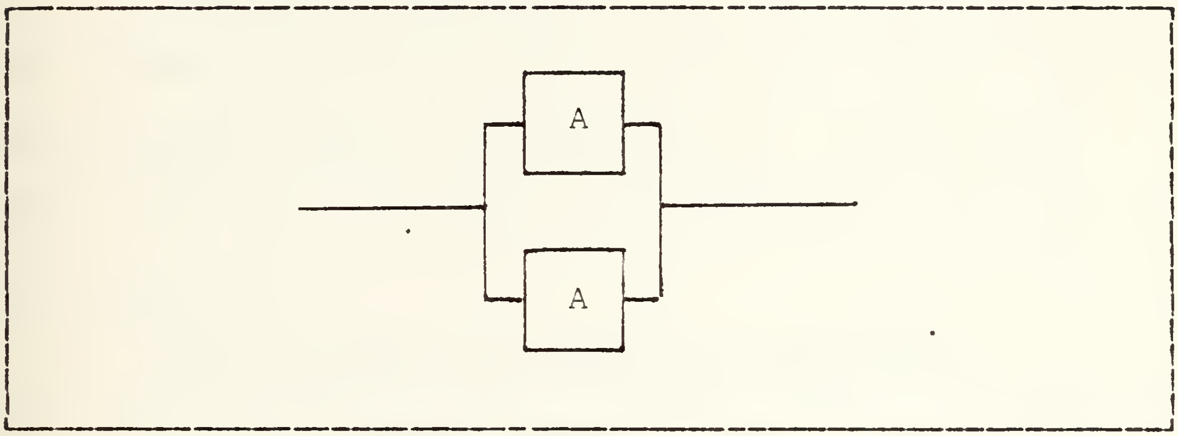



Figure 3: Two-Component Parallel System

Let

$$T_1 \sim EXP(\lambda) \; , \; T_2 \sim EXP(\lambda) \; .$$

Then

$$T = \max(T_1, T_2)$$

$$\bar{F}(t) = P(\max(T_1, T_2) > t)$$

$$= 1 - P(\max(T_1, T_2) \leqslant t)$$

$$= 1 - P(T_1 \leqslant t, T_2 \leqslant t)$$

Assuming independence of the two components,

$$\bar{F}(t) = 1 - P(T_1 \leqslant t)\, P(T_2 \leqslant t)$$
$$= 1 - F_1(t)\, F_2(t)$$
$$= 1 - (1-e^{-\lambda t})(1-e^{-\lambda t})$$
$$= 1 - (1-2e^{-\lambda t} + e^{-2\lambda t})$$
$$= 2e^{-\lambda t} - e^{-2\lambda t}.$$

The shorthand notation for the system is

$$\mathrm{EXP}(2\lambda) + \mathrm{EXP}(\lambda).$$

This follows intuition as the system has an EXP($2\lambda$) life to begin with and when one component fails it has an extra EXP($\lambda$) life due to the memoryless property of the exponential distribution.

3. Standby-System with Dissimilar Components

Suppose a system consists of two components, one active and one spare. The active component stays in service until it fails and then immediately is replaced by the spare.

Let the time to failure of the two components be $T_1 \sim \mathrm{EXP}(\lambda_1)$ and $T_2 \sim \mathrm{EXP}(\lambda_2)$ respectively.

Then the system time to failure is

$$T = T_1 + T_2$$

and the survival function of the system is

$$\bar{F}(t) = P(T > t)$$

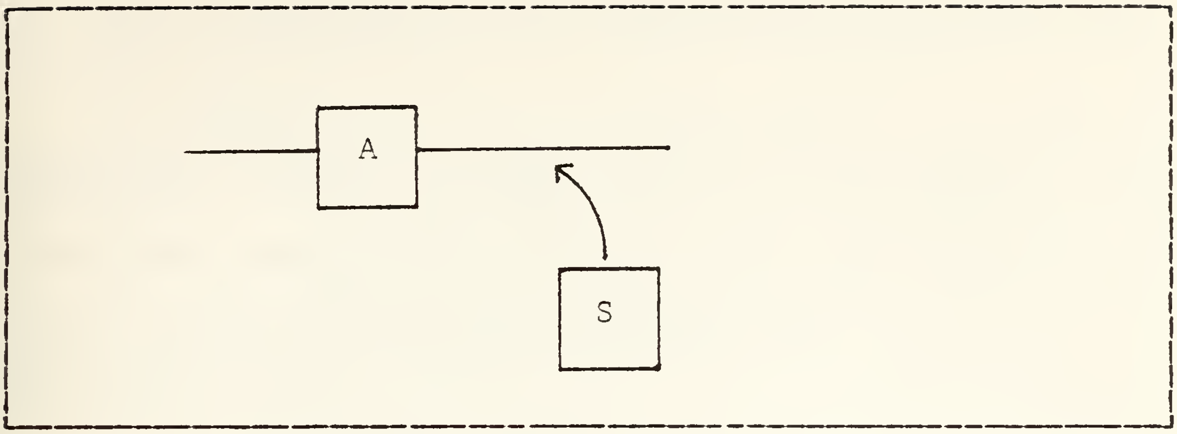



Figure 4: Standby System

$$= \bar{F}_1(t) + \int_0^t \bar{F}_2(t-s) f_1(s)\, ds$$

$$= e^{-\lambda_1 t} + \int_0^t e^{-\lambda_2 (t-s)} \lambda_1 e^{-\lambda_1 s}\, ds$$

$$= \frac{\lambda_1}{\lambda_1 - \lambda_2} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t}$$

The shorthand notation for the system's survival function should be obvious. The system has an EXP( $\lambda_1$) life from the active component and an additional EXP( $\lambda_2$) life from the spare. So the shorthand notation is

$$\text{EXP}(\lambda_1) + \text{EXP}(\lambda_2).$$

B. SUMS OF EXPONENTIALS WITH WEIGHT BETWEEN ZERO AND ONE

The examples given in the previous chapter only involved exponential lives with weight one. Now we will look at some structures, whose survival function has a shorthand notation which includes the MIX-notation and/or the ZERO-distribution.

1. Parallel System with Dissimilar Failure Rates

The notion of a parallel system has been introduced in Chapter III.A.2 . We now look at the case where

$$T_1 \sim EXP(\lambda_1) \text{ and } T_2 \sim EXP(\lambda_2).$$

Then

$$T = \max(T_1, T_2)$$

$$\begin{aligned}\overline{F}(t) &= P(\max(T_1, T_2) > t)\\ &= 1-P(\max(T_1, T_2) \leqslant t)\\ &= 1-P(T_1 \leqslant t, T_2 \leqslant t)\end{aligned}$$

Assuming independence of the two components

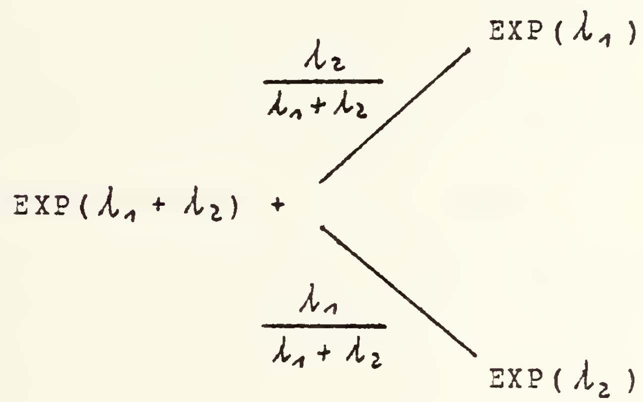
$$\begin{aligned}\overline{F}(t) &= 1-P(T_1 \leqslant t)\ P(T_2 \leqslant t)\\ &= 1-F_1(t)\ F_2(t)\\ &= 1-(1-e^{-\lambda_1 t})(1-e^{-\lambda_2 t})\\ &= 1-(1-e^{-\lambda_1 t}-e^{-\lambda_2 t}+e^{-(\lambda_1+\lambda_2)t})\\ &= e^{-\lambda_1 t}+e^{-\lambda_2 t}-e^{-(\lambda_1+\lambda_2)t}.\end{aligned}$$

To find the shorthand notation of the system consider all the ways which lead to the survival of the system:

- either both components survive
- or component 1 fails and component 2 survives
- or component 2 fails and component 1 survives.

If one component fails and one survives, in $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ fraction of the cases the survivor will be component 1 and in $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ fraction of the cases it will be component 2.

This can graphically be represented as




Making use of the MIX-notation the shorthand notation then is

$$\mathrm{EXP}(\lambda_1 + \lambda_2) + \mathrm{MIX}[\frac{\lambda_2}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_1), \frac{\lambda_1}{\lambda_1+\lambda_2}\mathrm{EXP}(\lambda_2)]$$

and using the distributive property it becomes

$$\mathrm{MIX}[\frac{\lambda_2}{\lambda_1 + \lambda_2}(\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_1 + \lambda_2)),$$
$$\frac{\lambda_1}{\lambda_1 + \lambda_2}(\mathrm{EXP}(\lambda_2) + \mathrm{EXP}(\lambda_1 + \lambda_2))].$$

As a check to see that this shorthand notation represents the survival function of the system, we derive the survival function from the shorthand notation:

$$\bar{F}(t) = \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-\lambda_1 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_1 e^{-\lambda_1 s}\,ds\right)$$

$$+ \frac{\lambda_1}{\lambda_1+\lambda_2}\left(e^{-\lambda_2 t} + \int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}\lambda_2 e^{-\lambda_2 s}\,ds\right)$$

$$= e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1+\lambda_2)t} .$$

This verifies that the shorthand notation indeed represents the system's survival function.

2. <u>Series System with One Spare</u>

Let us now look at a two-component series system, whose components have dissimilar failure rates with one component having a spare:

Component 1 has the constant failure rate $\lambda_1$ and component 2 and the spare have the constant failure rate $\lambda_2$. The spare can only replace component 2.

Let

$\bar{F}_1(t)$ : the survival function of component 1

$\bar{F}_2(t)$ : the survival function of the standby system component 2 with its spare.

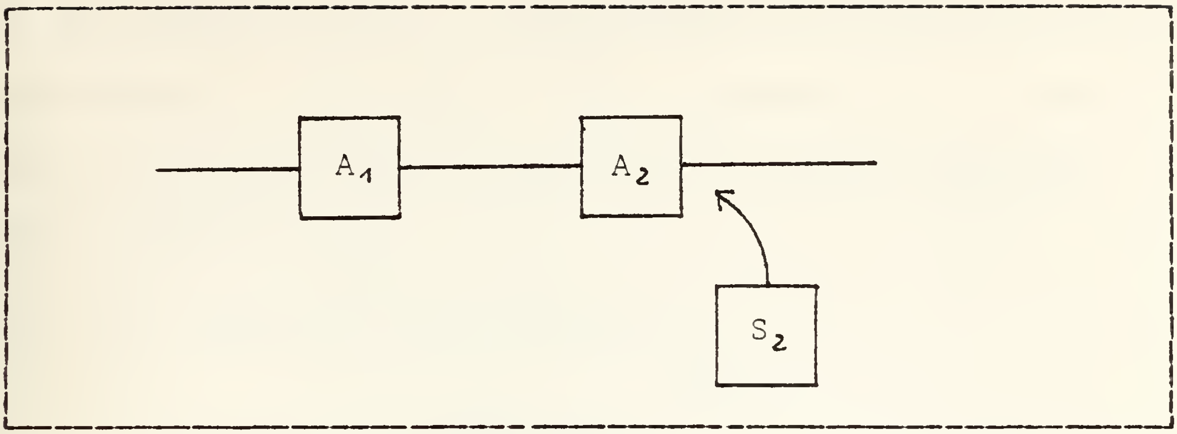



Figure 5: Series System with one Spare

The survival function for a standby system was derived in Chapter II.B. Therefore

$$\begin{aligned}\bar{F}_2(t) &= e^{-\lambda_2 t} + \int_0^t e^{-\lambda_2(t-s)} \lambda_2 e^{-\lambda_2 s}\, ds \\ &= e^{-\lambda_2 t} + \lambda_2 e^{-\lambda_2 t} \int_0^t ds \\ &= (1+\lambda_2 t)\, e^{-\lambda_2 t}.\end{aligned}$$

Now $\quad \bar{F}_1(t) = e^{-\lambda_1 t}$

Then $\quad \bar{F}(t) = \bar{F}_1(t)\, \bar{F}_2(t)$

$$= (1+\lambda_2 t)\, e^{-(\lambda_1+\lambda_2)t}.$$

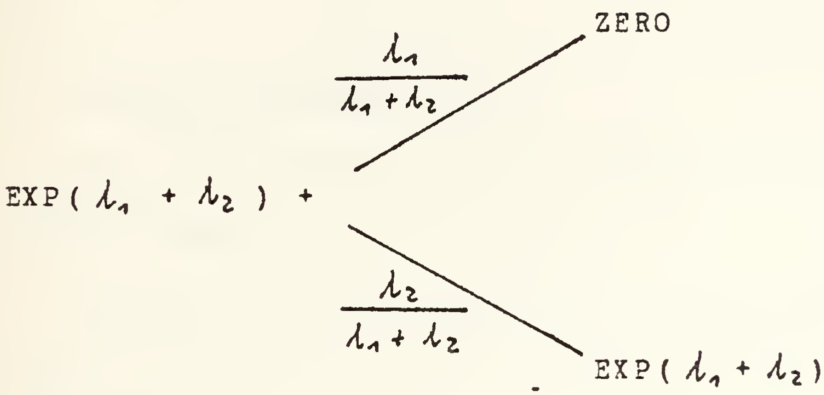
To translate the survival function into shorthand notation, let us consider the ways in which the system can survive:

-either both components survive

-or component 2 fails and its spare survives.

If one component fails, in $\frac{\lambda_1}{\lambda_1+\lambda_2}$ fraction of the time it will be component 1, which means that the system will not survive; in $\frac{\lambda_2}{\lambda_1+\lambda_2}$ fraction of the time the failing component will be component 2.

This can graphically be represented as




Using the MIX-notation the survival function then is

$$\text{EXP}(\lambda_1 + \lambda_2) + \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}\,\text{ZERO}, \frac{\lambda_2}{\lambda_1+\lambda_2}\text{EXP}(\lambda_1+\lambda_2)\right]$$

$$= \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{ZERO} + \text{EXP}(\lambda_1+\lambda_2)),\right.$$
$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\right]$$

$$= \text{MIX}\left[\frac{\lambda_1}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2)),\right.$$
$$\left.\frac{\lambda_2}{\lambda_1+\lambda_2}(\text{EXP}(\lambda_1+\lambda_2) + \text{EXP}(\lambda_1+\lambda_2))\right].$$

To prove, that the shorthand notation does represent the survival function, we derive the latter from the shorthand:

$$\bar{F}(t) = \frac{\lambda_1}{\lambda_1+\lambda_2}\, e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(e^{-(\lambda_1+\lambda_2)t} +\right.$$

$$\int_0^t e^{-(\lambda_1+\lambda_2)(t-s)}(\lambda_1+\lambda_2)e^{-(\lambda_1+\lambda_2)s}ds)$$

$$= (1+\lambda_2 t)\, e^{-(\lambda_1+\lambda_2)t} .$$

This is the previously found result and this verifies, that the shorthand notation does represent the system's survival function.

3. Two-out-of-Three System

As a last example in this chapter, we will look at a Two-out-of- Three system.

Consider a three component system, whose components have constant failure rates $\lambda_1$, $\lambda_2$ and $\lambda_3$ respectively. The system is functioning, as long as two out of three components are functioning (see Fig. 6).

In other words, the system is functioning as long as there is a path through the system .

Alternatively, the system can be visualized as a parallel-series system (compare Fig. 7).

The survival function of the system is

$$\begin{aligned}\bar{F}(t) &= P(T_1 >t \wedge T_2 >t) + P(T_1 >t \wedge T_3 >t)\\ &+ P(T_2 >t \wedge T_3 >t)\\ &- P((T_1 >t \wedge T_2 >t) \wedge (T_1 >t \wedge T_3 >t))\\ &- P((T_1 >t \wedge T_2 >t) \wedge (T_2 >t \wedge T_3 >t))\\ &- P((T_1 >t \wedge T_3 >t) \wedge (T_2 >t \wedge T_3 >t))\end{aligned}$$

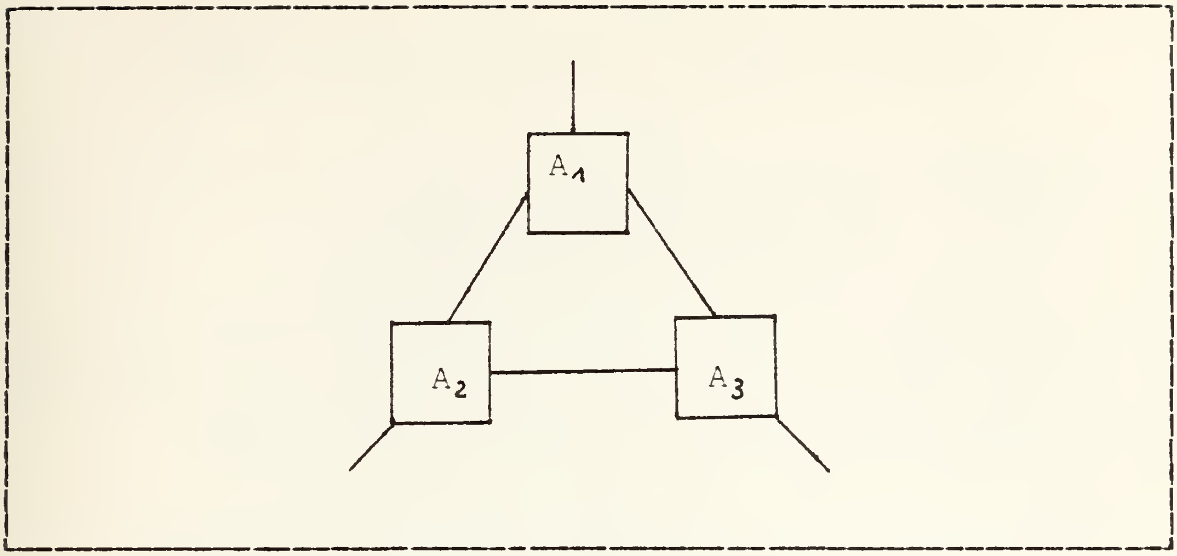



Figure 6: Two-out-of-Three System

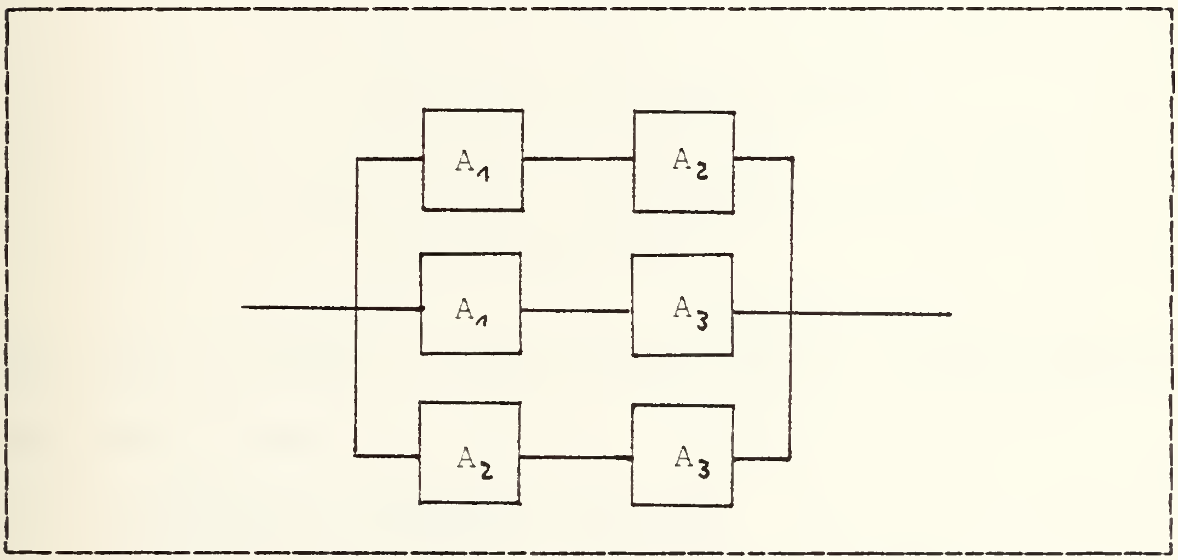



Figure 7: Two-out-of-Three System

$$+ P((T_1 > t \wedge T_2 > t) \wedge (T_1 > t \wedge T_3 > t)$$

$$\wedge P(T_2 > t \wedge T_3 > t)).$$

Thus

$$\bar{F}(t) = P(T_1 > t \wedge T_2 > t) + P(T_1 > t \wedge T_3 > t)$$
$$+ P(T_2 > t \wedge T_3 > t)$$
$$- 3P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$
$$+ P(T_1 > t \wedge T_2 > t \wedge T_3 > t)$$

Therefore, and assuming independence of the components,

$$\bar{F}(t) = P(T_1 > t)\ P(T_2 > t) + P(T_1 > T)\ P(T_3 > t)$$
$$+ P(T_2 > t)\ P(T_3 > t)$$
$$- 3P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$
$$+ P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$

$$= P(T_1 > t)\ P(T_2 > t) + P(T_1 > t)\ P(T_3 > t)$$
$$+ P(T_2 > t)\ P(T_3 > t)$$
$$- 2P(T_1 > t)\ P(T_2 > t)\ P(T_3 > t)$$

$$= e^{-(\lambda_1+\lambda_2)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_2+\lambda_3)t}$$
$$- 2e^{-(\lambda_1+\lambda_2+\lambda_3)t} .$$

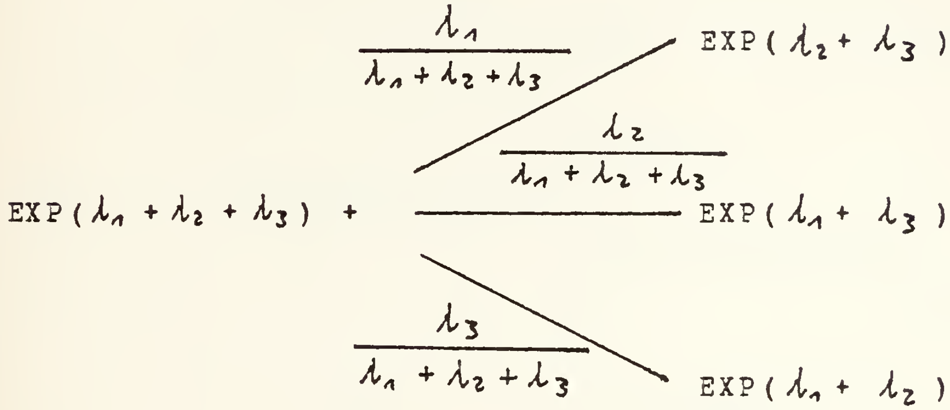
Now let us consider all the possible ways, in which the system can survive:

- either all components survive
- or component 1 fails and component 2 and 3 survive
- or component 2 fails and component 1 and 3 survive

- or component 3 fails and component 1 and 2 survive.

If a component fails and the other two survive, in $\frac{\lambda_i}{\lambda_1+\lambda_2+\lambda_3}$ fraction of the time it will be component i , i = 1,2,3 .

This can graphically be represented as




The shorthand notation then is

$$EXP(\lambda_1+\lambda_2+\lambda_3) + MIX[\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}EXP(\lambda_2+\lambda_3), \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}EXP(\lambda_1+\lambda_3), \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}EXP(\lambda_1+\lambda_2)],$$

$$= MIX[\frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}(EXP(\lambda_2+\lambda_3) + EXP(\lambda_1+\lambda_2+\lambda_3)), \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}(EXP(\lambda_1+\lambda_3) + EXP(\lambda_1+\lambda_2+\lambda_3)), \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}(EXP(\lambda_1+\lambda_2) + EXP(\lambda_1+\lambda_2+\lambda_3))].$$

Again, as a check that the shorthand notation represents the survival function, let us derive the survival function from the shorthand notation:

$$F(t) = \frac{\lambda_1}{\lambda_1+\lambda_2+\lambda_3}\Big[ e^{-(\lambda_2+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_2+\lambda_3) e^{-(\lambda_2+\lambda_3)s} ds \Big]$$

$$+ \frac{\lambda_2}{\lambda_1+\lambda_2+\lambda_3}\Big[ e^{-(\lambda_1+\lambda_3)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_1+\lambda_3) e^{-(\lambda_1+\lambda_3)s} ds \Big]$$

$$+ \frac{\lambda_3}{\lambda_1+\lambda_2+\lambda_3}\Big[ e^{-(\lambda_1+\lambda_2)t} + \int_0^t e^{-(\lambda_1+\lambda_2+\lambda_3)(t-s)} (\lambda_1+\lambda_2) e^{-(\lambda_1+\lambda_2)s} ds \Big]$$

$$= e^{-(\lambda_2+\lambda_3)t} + e^{-(\lambda_1+\lambda_3)t} + e^{-(\lambda_1+\lambda_2)t} - 2 e^{-(\lambda_1+\lambda_2+\lambda_3)t}$$

The result again proves that the shorthand notation indeed represents the survival function of the system.

# IV. IMPLEMENTING THE SHORTHAND ON THE TI-59

The concept of a reliability shorthand is introduced in the course "Reliability and Weapons System Effectiveness Measurements", OA 4302, at the Naval Postgraduate School, Monterey. Most students taking the course are in the Operations Research (OR) - Curriculum.

The choice of the TI-59 as the computing device, on which the shorthand was to be implemented, was based on the fact, that each student in the OR-Curriculum is issued a TI-59 for use in basic probability and statistics courses. Thus almost every student at the Naval Postgraduate School, who is introduced to the shorthand, is familiar with the TI-59 and has access to such a calculator.

A program, that uses the shorthand notation, times to failure and failure rates as input, should

- calculate the survival probability of basic structures / small systems and
- require moderate computation time.

To achieve these requirements it was decided to incorporate all solutions for the convolution of up to four exponential random variables in the program. The formulas that were used are given in Appendix A.

Two programs are provided in this paper.

Program 1 can be used when all rates are dissimilar or all are the same. It uses the formulas on pages 37 and 38 only.

Program 2 can be used for the general case. It makes use of all the formulas given in Appendix A. The program includes a sorting routine that determines the applicable formula from the entered failure rates.

A user guide to the two programs is provided in Appendix B.

## V. SUMMARY

There is a reliability shorthand that denotes the survival function of a system, assuming that the failure rates of all components are constant.

This shorthand can be implemented on the TI-59 handheld calculator. With failure rates, time to failure and shorthand as input the TI-59 calculates the survival probability of the system.

Knowledge of calculus is not necessary to use this method, whereas the standard procedure, finding the survival probability by convolution, requires knowledge of integral calculus.

The choice of the TI-59 as the computing device for the implementation of the shorthand, though, implied limitations; the number of failure rates is limited due to the limited storage capacity of the TI-59, and computing times are comparatively long. The TI-59 can therefore only be used for smaller systems, preferably for the solution of classroom problems.

For the solution of larger problems, the shorthand should be implemented on a state-of-the-art personal

computer using a general algorithm for the convolution of any number of exponential random variables.

# APPENDIX A

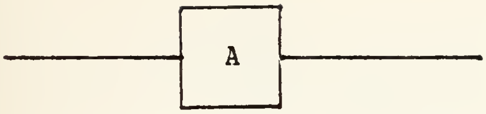
## CONVOLUTION FORMULAS

Appendix A contains formulas for the convolution of up to four exponential random variables.

For the two special cases, when all random variables have the same failure rate and all have different failure rates, general formulas for the convolution of any number of exponential random variables are given.

These formulas are used in the two TI-59 programs provided in Appendix B.

System:



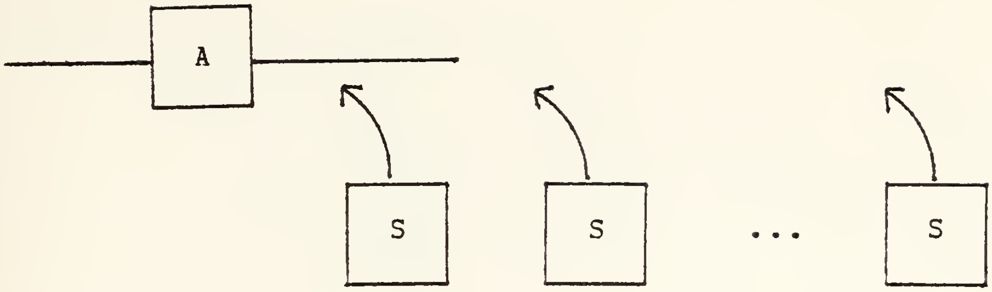
Shorthand: EXP($\lambda$)

Survival Function: $\bar{F}(t) = e^{-\lambda t}$

Description:

A single active component with constant failure rate $\lambda$.

System:




Shorthand: EXP( $\lambda$ ) + EXP( $\lambda$ ) + ... + EXP( $\lambda$ )

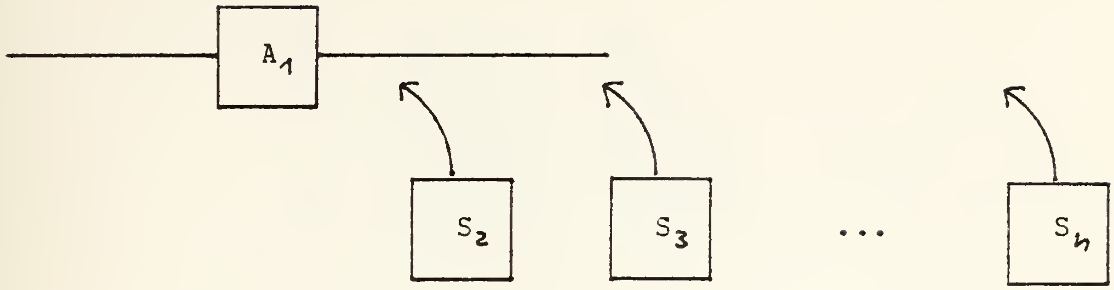
Survival Function: $\bar{F}(t) = \left( \frac{(\lambda t)^0}{0!} + \frac{(\lambda t)^1}{1!} + \cdots + \frac{(\lambda t)^{n-1}}{(n-1)!} \right) e^{-\lambda t}$

$$= \sum_{i=1}^{n} \frac{(\lambda t)^{i-1}}{(i-1)!} e^{-\lambda t}$$

Description:

A single active component with constant failure rate is supported by n-1 identical spares.

System:



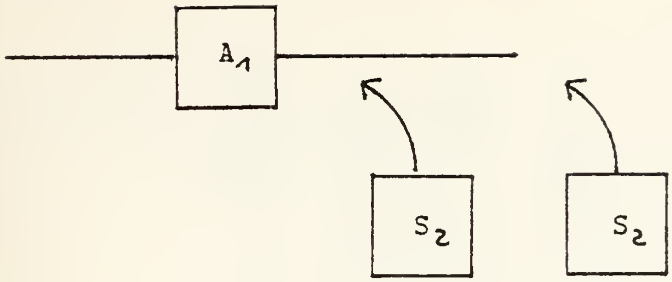

Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + \ldots + EXP(\lambda_n)$

Survival Function: $$\bar{F}(t) = \sum_{i=1}^{n} \left( \prod_{j \neq i} \frac{\lambda_j}{\lambda_j - \lambda_i} e^{-\lambda_i t} \right)$$

Description:

A single active component with constant failure rate is supported by n-1 spares. The active component and the spares have all constant, but dissimilar failure rates.

System:




Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\overline{F}(t) = Ae^{-\lambda_1 t} + (B + Ct)\, e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^{\,2}}{(\lambda_2 - \lambda_1)^2}$

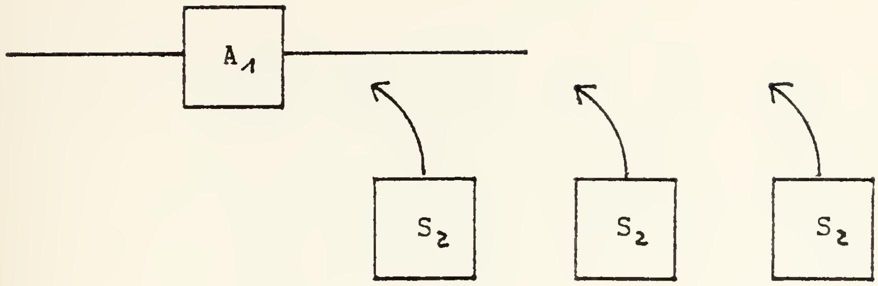
$B = 1 - A$

$C = \dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

Description:

A single active component with constant failure rate $\lambda_1$ is supported by two spares with identical constant failure rate $\lambda_2$.

System:




Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + (B + Ct + Dt^2)e^{-\lambda_2 t}$

where $$A = \frac{\lambda_2^3}{(\lambda_2 - \lambda_1)^3}$$

$$B = 1 - A$$

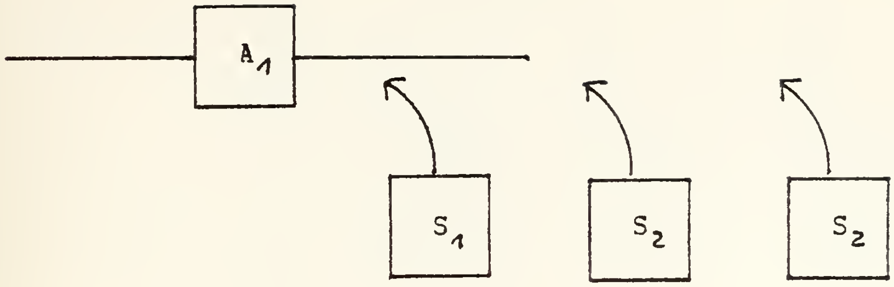
$$C = \lambda_2 - \frac{\lambda_2^3}{(\lambda_1 - \lambda_2)^2}$$

$$D = \frac{\lambda_1 \lambda_2^2}{2(\lambda_1 - \lambda_2)}$$

Description:

A single active component with constant failure rate $\lambda_1$ is supported by three spares with identical constant failure rate $\lambda_2$.

System:




Shorthand: $EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Survival Function: $F(t) = (A + Bt)e^{-\lambda_1 t} + (C + Dt)e^{-\lambda_2 t}$

where $A = \dfrac{\lambda_2^3 - 3\lambda_2^2\lambda_1}{(\lambda_2 - \lambda_1)^3}$

$B = \dfrac{\lambda_1 \lambda_2^2}{(\lambda_2 - \lambda_1)^2}$

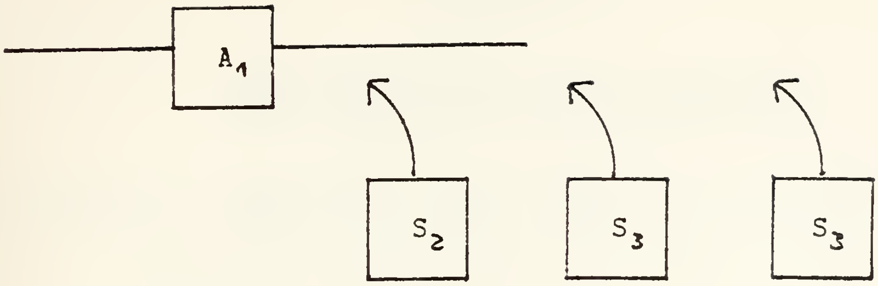
$C = 1 - A$

$D = \dfrac{\lambda_1^2 \lambda_2}{(\lambda_1 - \lambda_2)^2}$

Description:

A single active component with constant failure rate $\lambda_1$ is supported by one identical spare and two spares with dissimilar, constant failure rate $\lambda_2$.

System:




Shorthand: $EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3) + EXP(\lambda_3)$

Survival Function: $\bar{F}(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_2 t} + (C + Dt)e^{-\lambda_3 t}$

where
$$A = \frac{\lambda_2 \lambda_3^2}{(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)^2}$$

$$B = \frac{\lambda_1 \lambda_3^2}{(\lambda_1-\lambda_2)(\lambda_3-\lambda_2)^2}$$

$$C = \frac{\lambda_1 \lambda_2}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)} + \frac{\lambda_1\lambda_2\lambda_3}{(\lambda_1-\lambda_2)}\left(\frac{1}{(\lambda_1-\lambda_3)^2} - \frac{1}{(\lambda_2-\lambda_3)^2}\right)$$

$$D = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_3)}$$

Description:

A single active component with constant failure rate $\lambda_1$ has three spares. One spare has constant failure rate $\lambda_2$, two spares are identical with constant failure rate $\lambda_3$.

# APPENDIX B

## USER GUIDE TO TI-59 PROGRAMS

Appendix B contains a user guide to two TI-59 programs, which use reliability shorthand and failure rates as input to compute the survival probability of a system.

PROGRAM 1 is designed for the two special cases where the reliability shorthand is of the form

$$\mathrm{EXP}(\lambda) + \mathrm{EXP}(\lambda) + \ldots + \mathrm{EXP}(\lambda)$$

or

$$\mathrm{EXP}(\lambda_1) + \mathrm{EXP}(\lambda_2) + \ldots + \mathrm{EXP}(\lambda_n).$$

In the first case the number of terms is not limited, whereas in the second case the number of terms is limited to 40 due to limited storage capacity of the TI-59. In this case the number of terms can be increased to 70 by entering 9 in the display and pressing 2nd Op 1 7 .

PROGRAM 2 is designed to solve problems of the kind, that were introduced in Chapter III.B. . Due to limited memory of the TI-59 the number of exponential terms under one weight in shorthand notation is limited to four.

All results will be printed, if the TI-59 is connected to a TI PC-100A or TI PC-100C printer.

PROGRAM 1 : Procedure

1. Use any library module. Read in program 1 (side 1 of the magnetic card)
2. Press 2nd C' to initialize.
3. Enter n, the number of exponential terms to be convolved, in the display and press A.
4. Enter time t and press B.
5. Enter $\lambda_i$ and press C . When all failure rates are the same, enter $\lambda$ only once.
6. a) To find the survival probability of the system, when all failure rates are the same, press 2nd A'.

   b) To find the survival probability of the system, when all failure rates are dissimilar, press 2nd B'.

PROGRAM 1 : Sample Problems

1. Find the survival probability of a parallel system

   ( compare Chapter III.A. 2 )

   a) $\lambda = .3$ , $t = 7$ , $n = 2$

   b) Shorthand notation:

      EXP(.6) + EXP(.3)

   c)

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 2 | n | A | 0 |
| 7 | t | B | 7 |
| .6 | $2\lambda$ | C | .3 |
| .3 | $\lambda$ | C | .3 |
| | $\bar{F}(t)$ | B' | .2299172797 |

calculation takes 13 seconds

2. Find the survival probability of a standby-system with dissimilar components ( compare Chapter III.A.3 ) .

a) $\lambda_1 = .4$ , $\lambda_2 = .5$ , $t = 6$ , $n = 2$

b) Shorthand notation:

EXP(.4) + EXP(.5)

c)

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 2 | $n$ | A | 0 |
| 6 | $t$ | B | 6 |
| .4 | $\lambda_1$ | C | .4 |
| .5 | $\lambda_2$ | C | .5 |
| | $\overline{F}(t)$ | B' | .254441493 |

calculation takes 13 seconds

3. Find the survival probability of a standby-system with one active component and four similar spares.

a) $\lambda = .3$ , $t = 7$ , $n = 5$

b) Shorthand notation:

EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3) + EXP(.3)

c)

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 5 | n | A | 0 |
| 7 | t | B | 7 |
| .3 | $\lambda$ | C | .3 |
| | $\bar{F}(t)$ | A' | .9378738848 |

calculation takes 9 seconds

PROGRAM 2 : Procedure

CASE I : To find the convolution of up to four exponential random variables.

1. Use any library module.

   Re-Partition ( enter 2 in the display, press 2nd Op 17 ).

   Read in all four sides of the magnetic card.

2. Press 2nd C' to initialize.

3. Enter n, the number of exponential terms to be convolved, in the display and press A.

4. Enter time t and press B.

5. Enter $\lambda_i$ and press C ( n entries ) .

   REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

6. To find the survival probability of the system press E.

PROGRAM 2, CASE I : Sample Problems

(1) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .3$ , $\lambda_2 = .4$ , $t = 7$

Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 3 | n | A | 0 |
| 7 | t | B | 7 |
| .3 | $\lambda_1$ | C | .3 |
| .4 | $\lambda_2$ | C | .4 |
| .4 | $\lambda_2$ | C | .4 |
| | $\bar{F}(t)$ | E | .5363473866 |

calculation takes 14 seconds

(2) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .2$ , $\lambda_2 = .4$ , $t = 3$

Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 4 | n | A | 0 |
| 3 | t | B | 3 |
| .2 | $\lambda_1$ | C | .2 |
| .4 | $\lambda_2$ | C | .4 |
| .4 | $\lambda_2$ | C | .4 |
| .4 | $\lambda_2$ | C | .4 |
| | $\bar{F}(t)$ | E | .9809746099 |

calculation takes 20 seconds

(3) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_2)$

Sample values : $\lambda_1 = .4$ , $\lambda_2 = .3$ , $t = 5$

Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 4 | n | A | 0 |
| 5 | t | B | 5 |
| .4 | $\lambda_1$ | C | .4 |
| .4 | $\lambda_1$ | C | .4 |
| .3 | $\lambda_2$ | C | .3 |
| .3 | $\lambda_2$ | C | .3 |
| | $\bar{F}(t)$ | E | .9029040721 |

calculation takes 20 seconds

(4) Shorthand notation

$EXP(\lambda_1) + EXP(\lambda_2) + EXP(\lambda_3) + EXP(\lambda_3)$

Sample values : $\lambda_1 = .1$ , $\lambda_2 = .3$ , $\lambda_3 = .5$ ,

$t = 10$

Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 4 | n | A | 0 |
| 10 | t | B | 10 |
| .1 | $\lambda_1$ | C | .1 |
| .3 | $\lambda_2$ | C | .3 |
| .5 | $\lambda_3$ | C | .5 |
| .5 | $\lambda_3$ | C | .5 |
| | $\bar{F}(t)$ | E | .7312684703 |

calculation takes 25 seconds

PROGRAM 2 : Procedure

CASE II : to solve problems of the kind, that were introduced in Chapter III.B. .

1. Derive the system's shorthand notation. Find either the
   - graphical representation or
   - the MIX-notation .
2. Use any library module.

   Re-Partition ( enter 2 in the display, press 2nd Op 17 ).

   Read in all four sides of the magnetic card.
3. Press 2nd C' to initialize.
4. Enter time t and press B.
5. Repeat the following steps for each path of the graphical representation, i.e. for each convolution in the MIX-notation.

   a) Enter n, the number of exponential terms to be convolved, in the display and press A.

   b) Enter $\lambda_i$ and press C.

      REMARK: Failure rates, which appear only once in the expression, have to be entered before failure rates, that appear several times.

   c) Enter $p_i$, the weight in the ith path, and press D.

   d) To find the part of the system's survival probability, that is contributed by the ith path, press E.

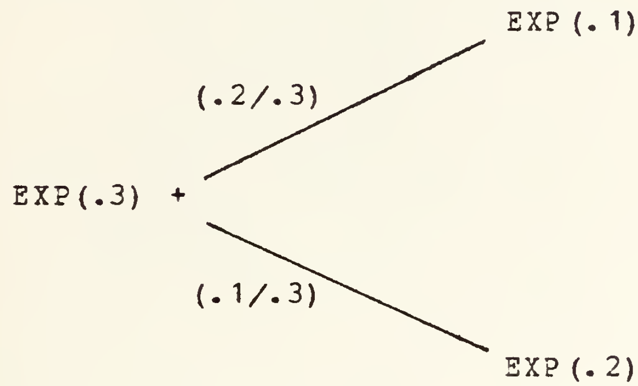
6. To find the survival probability of the system press 2nd E'.

PROGRAM 2, CASE II : Sample Problems

1. Find the survival probability of a parallel system with dissimilar failure rates (compare Chapter III.B.1).

   a) $\lambda_1 = .1$ , $\lambda_2 = .2$ , $t = 2$

   b) Shorthand notation




$$\overline{F}(t) = \text{MIX}[\ (.2/.3)(\ \text{EXP}(.1) + \text{EXP}(.3)),$$
$$(.1/.3)(\ \text{EXP}(.2) + \text{EXP}(.3)\ ].$$

c)

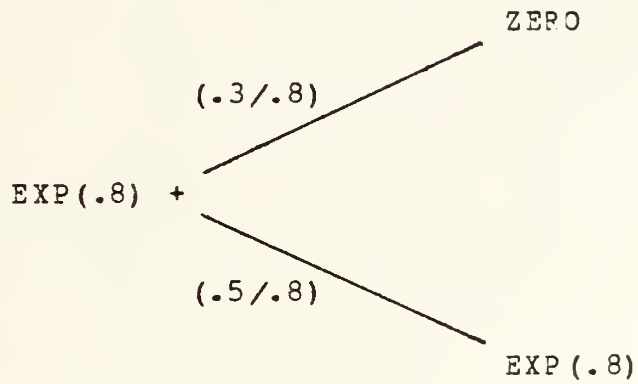
Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 2 | $t$ | B | 2 |
| 2 | $n_1$ | A | 0 |
| .1 | $\lambda_1$ | C | .1 |
| .3 | $\lambda_1 + \lambda_2$ | C | .3 |
| (.2/.3) | $p_1$ | D | .6666666667 |
| | | E | .635793541 |
| 2 | $n_2$ | A | 0 |
| .2 | $\lambda_2$ | C | .2 |
| .3 | $\lambda_1 + \lambda_2$ | C | .3 |
| (.1/.3) | $p_2$ | D | .3333333333 |
| | | E | .304445622 |
| | $\overline{F}(t)$ | E' | .940239163 |

2. Find the survival probability of a series system with one spare as introduced in Chapter III.B.2 .

a) $\lambda_1 = .3$ , $\lambda_2 = .5$ , $t = 7$

b) Shorthand notation




$\bar{F}(t)$ = MIX[ (.3/.8)( EXP(.8),

(.5/.8)( EXP(.8) + EXP(.8) ].

c)

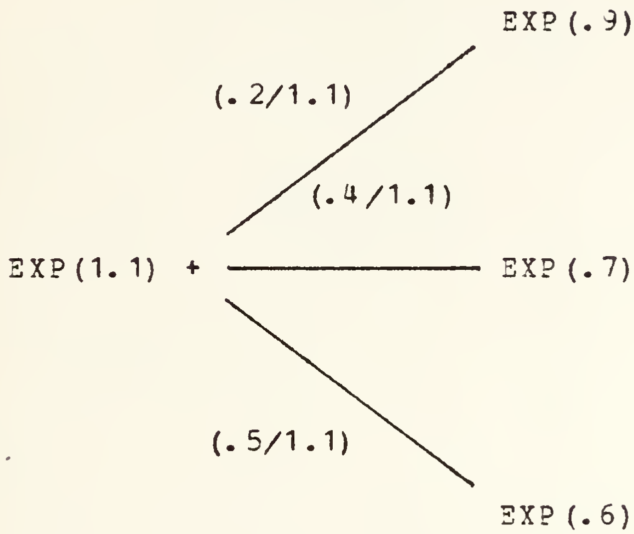
Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 7 | t | B | 7 |
| 1 | $n_1$ | A | 0 |
| .8 | $\lambda_1 + \lambda_2$ | C | .8 |
| (.3/.8) | $p_1$ | D | .375 |
| | | E | .0013866989 |
| 2 | $n_2$ | A | 0 |
| .8 | $\lambda_1 + \lambda_2$ | C | .8 |
| .8 | $\lambda_1 + \lambda_2$ | C | .8 |
| (.5/.8) | $p_2$ | D | .625 |
| | | E | .0152536878 |
| | $\overline{F}(t)$ | E' | .0166403867 |

3. Find the survival probability of a Two-out-of-Three System as introduced in Chapter III.B.3 .

a) $\lambda_1 = .2$ , $\lambda_2 = .4$ , $\lambda_3 = .5$ , $t = 9$

b) Shorthand notation




$\bar{F}(t)$ = MIX[ (.2/1.1) ( EXP(.9) + EXP(1.1)),

(.4/1.1) ( EXP(.7) + EXP(1.1)),

(.5/1.1) ( EXP(.6) + EXP(1.1)) ].

c)

Procedure :

| Enter | Comment | Press | Display |
|---|---|---|---|
| | Initialize | C' | 0 |
| 9 | $t$ | B | 9 |
| 2 | $n_1$ | A | 0 |
| 1.1 | $\lambda_1 + \lambda_2 + \lambda_3$ | C | 1.1 |
| .9 | $\lambda_2 + \lambda_3$ | C | .9 |
| (.2/1.1) | $p_1$ | D | .1818181818 |
| | | E | .0002624871 |
| 2 | $n_2$ | A | 0 |
| 1.1 | $\lambda_1 + \lambda_2 + \lambda_3$ | C | 1.1 |
| .7 | $\lambda_1 + \lambda_3$ | C | .7 |
| (.4/1.1) | $p_2$ | D | .3636363636 |
| | | E | .0018043754 |
| 2 | $n_3$ | A | 0 |
| 1.1 | $\lambda_1 + \lambda_2 + \lambda_3$ | C | 1.1 |
| .6 | $\lambda_1 + \lambda_2$ | C | .6 |
| (.5/1.1) | $p_3$ | D | .4545454545 |
| | | E | .0044892129 |
| | $\overline{F}(t)$ | E' | .0065560755 |

**PROGRAM 1**

```
000  91  R/S
001  76  LBL
002  18  C'
003  29  CP
004  25  CLR
005  47  CMS
006  81  RST
007  76  LBL
008  11   A
009  42  STO
010  00   00
011  75   -
012  01   1
013  95   =
014  42  STO
015  09   09
016  29  CP
017  02   2
018  00   0
019  42  STO
020  08   08
021  01   1
022  42  STO
023  17   17
024  00   0
025  42  STO
026  18   18
027  91  R/S
028  76  LBL
029  12   B
030  42  STO
031  01   01
032  91  R/S
033  76  LBL
034  13   C
035  72  ST*
036  08   08
037  69  OP
038  28   28
039  91  R/S
040  76  LBL
041  16  A'
042  43  RCL
043  20   20
044  65   ×
045  43  RCL
046  01   01
047  95   =
048  42  STO
049  02   02
050  00   0
051  01   1
052  42  STO
053  04   04
054  42  STO
055  05   05
056  42  STO
057  18   18
058  76  LBL
059  28  LOG
060  29  CP
061  43  RCL
062  09   09
063  67   EQ
064  29  CP
065  43  RCL
066  02   02
067  45  Y^X
068  43  RCL
069  05   05
070  55   ÷
071  43  RCL
072  04   04
073  95   =
074  44  SUM
075  18   18
076  69  OP
077  39   39
078  69  OP
079  25   25
080  43  RCL
081  05   05
082  49  PRD
083  04   04
084  61  GTO
085  28  LOG
086  76  LBL
087  29  CP
088  43  RCL
089  02   02
090  94  +/-
091  22  INV
092  23  LNX
093  65   ×
094  71  SBR
095  47  CMS
096  95   =
097  99  PRT
098  91  R/S
099  76  LBL
100  17  B'
101  02   2
102  00   0
103  42  STO
104  08   08
105  76  LBL
106  39  COS
107  02   2
108  00   0
109  42  STO
110  06   06
111  01   1
112  42  STO
113  16   16
114  76  LBL
115  38  SIN
116  29  CP
117  73  RC*
118  06   06
119  67   EQ
```

PROGRAM 1 continued

```
120  36 PGM
121  73 RC*
122  08  08
123  32 X:T
124  73 RC*
125  06  06
126  22 INV
127  67  EQ
128  30 TAN
129  69 OP
130  26  26
131  61 GTO
132  38 SIN
133  76 LBL
134  30 TAN
135  49 PRD
136  16  16
137  75  -
138  73 RC*
139  08  08
140  95  =
141  35 1/X
142  49 PRD
143  16  16
144  43 RCL
145  00  00
146  32 X:T
147  43 RCL
148  06  06
149  75  -
150  01  1
151  09  9
152  95  =
153  77  GE
154  36 PGM
155  69 OP
156  26  26
157  61 GTO
158  38 SIN
159  76 LBL
160  36 PGM
161  73 RC*
162  08  08
163  94 +/-
164  65  ×
165  43 RCL
166  01  01
167  95  =
168  22 INV
169  23 LNX
170  65  ×
171  43 RCL
172  16  16
173  95  =
174  44 SUM
175  18  18
176  43 RCL
177  00  00
178  32 X:T
179  43 RCL
180  08  08
181  75  -
182  01  1
183  09  9
184  95  =
185  67  EQ
186  37 P/R
187  69 OP
188  28  28
189  61 GTO
190  39 COS
191  76 LBL
192  37 P/R
193  71 SBR
194  47 CMS
195  99 PRT
196  91 R/S
197  76 LBL
198  47 CMS
199  43 RCL
200  18  18
201  92 RTN
```

LABEL ADRESSES

```
002  18 C'
008  11  A
029  12  B
034  13  C
041  16 A'
059  28 LOG
087  29 CP
100  17 B'
106  39 COS
115  38 SIN
134  30 TAN
160  36 PGM
192  37 P/R
198  47 CMS
```

PROGRAM 2

```
000  86  STF
001  00   00
002  61  GTO
003  15   E
004  76  LBL
005  18  C'
006  29  CP
007  25  CLR
008  47  CMS
009  91  R/S
010  76  LBL
011  11   A
012  42  STO
013  00   00
014  75   -
015  01   1
016  95   =
017  42  STO
018  09   09
019  29  CP
020  01   1
021  00   0
022  42  STO
023  08   08
024  01   1
025  42  STO
026  17   17
027  00   0
028  42  STO
029  18   18
030  91  R/S
031  76  LBL
032  12   B
033  42  STO
034  01   01
035  91  R/S
036  76  LBL
037  13   C
038  72  ST*
039  08   08
040  69  OP
041  28   28
042  91  R/S
043  76  LBL
044  14   D
045  49  PRD
046  17   17
047  91  R/S
048  76  LBL
049  16  A'
050  43  RCL
051  10   10
052  65   x
053  43  RCL
054  01   01
055  95   =
056  42  STO
057  02   02
058  00   0
059  01   1
060  42  STO
061  04   04
062  42  STO
063  05   05
064  42  STO
065  18   18
066  76  LBL
067  28  LOG
068  29  CP
069  43  RCL
070  09   09
071  67   EQ
072  29  CP
073  43  RCL
074  02   02
075  45  YX
076  43  RCL
077  05   05
078  55   ÷
079  43  RCL
080  04   04
081  95   =
082  44  SUM
083  18   18
084  69  OP
085  39   39
086  69  OP
087  25   25
088  43  RCL
089  05   05
090  49  PRD
091  04   04
092  61  GTO
093  28  LOG
094  76  LBL
095  29  CP
096  43  RCL
097  02   02
098  94  +/-
099  22  INV
100  23  LNX
101  65   x
102  71  SBR
103  47  CMS
104  95   =
105  91  R/S
106  76  LBL
107  17  B'
108  01   1
109  00   0
110  42  STO
111  08   08
112  76  LBL
113  39  COS
114  01   1
115  00   0
116  42  STO
117  06   06
118  01   1
119  42  STO
```

PROGRAM 2 continued

| Step | Code | Key |
|---|---|---|
| 120 | 16 | 16 |
| 121 | 76 | LBL |
| 122 | 38 | SIN |
| 123 | 29 | CP |
| 124 | 73 | RC* |
| 125 | 06 | 06 |
| 126 | 67 | EQ |
| 127 | 36 | PGM |
| 128 | 73 | RC* |
| 129 | 08 | 08 |
| 130 | 32 | X:T |
| 131 | 73 | RC* |
| 132 | 06 | 06 |
| 133 | 22 | INV |
| 134 | 67 | EQ |
| 135 | 30 | TAN |
| 136 | 69 | OP |
| 137 | 26 | 26 |
| 138 | 61 | GTO |
| 139 | 38 | SIN |
| 140 | 76 | LBL |
| 141 | 30 | TAN |
| 142 | 49 | PRD |
| 143 | 16 | 16 |
| 144 | 75 | - |
| 145 | 73 | RC* |
| 146 | 08 | 08 |
| 147 | 95 | = |
| 148 | 35 | 1/X |
| 149 | 49 | PRD |
| 150 | 16 | 16 |
| 151 | 43 | RCL |
| 152 | 00 | 00 |
| 153 | 32 | X:T |
| 154 | 43 | RCL |
| 155 | 06 | 06 |
| 156 | 75 | - |
| 157 | 09 | 9 |
| 158 | 95 | = |
| 159 | 77 | GE |
| 160 | 36 | PGM |
| 161 | 69 | OP |
| 162 | 26 | 26 |
| 163 | 61 | GTO |
| 164 | 38 | SIN |
| 165 | 76 | LBL |
| 166 | 36 | PGM |
| 167 | 73 | RC* |
| 168 | 08 | 08 |
| 169 | 94 | +/- |
| 170 | 65 | × |
| 171 | 43 | RCL |
| 172 | 01 | 01 |
| 173 | 95 | = |
| 174 | 22 | INV |
| 175 | 23 | LNX |
| 176 | 65 | × |
| 177 | 43 | RCL |
| 178 | 16 | 16 |
| 179 | 95 | = |
| 180 | 44 | SUM |
| 181 | 18 | 18 |
| 182 | 43 | RCL |
| 183 | 00 | 00 |
| 184 | 32 | X:T |
| 185 | 43 | RCL |
| 186 | 08 | 08 |
| 187 | 75 | - |
| 188 | 09 | 9 |
| 189 | 95 | = |
| 190 | 67 | EQ |
| 191 | 37 | P/R |
| 192 | 69 | OP |
| 193 | 28 | 28 |
| 194 | 61 | GTO |
| 195 | 39 | COS |
| 196 | 76 | LBL |
| 197 | 37 | P/R |
| 198 | 71 | SBR |
| 199 | 47 | CMS |
| 200 | 91 | R/S |
| 201 | 76 | LBL |
| 202 | 47 | CMS |
| 203 | 43 | RCL |
| 204 | 18 | 18 |
| 205 | 65 | × |
| 206 | 43 | RCL |
| 207 | 17 | 17 |
| 208 | 95 | = |
| 209 | 99 | PRT |
| 210 | 44 | SUM |
| 211 | 19 | 19 |
| 212 | 92 | RTN |
| 213 | 76 | LBL |
| 214 | 22 | INV |
| 215 | 43 | RCL |
| 216 | 11 | 11 |
| 217 | 33 | X² |
| 218 | 55 | ÷ |
| 219 | 53 | ( |
| 220 | 43 | RCL |
| 221 | 11 | 11 |
| 222 | 75 | - |
| 223 | 43 | RCL |
| 224 | 10 | 10 |
| 225 | 54 | ) |
| 226 | 33 | X² |
| 227 | 95 | = |
| 228 | 42 | STO |
| 229 | 07 | 07 |
| 230 | 65 | × |
| 231 | 53 | ( |
| 232 | 43 | RCL |
| 233 | 10 | 10 |
| 234 | 94 | +/- |
| 235 | 65 | × |
| 236 | 43 | RCL |
| 237 | 01 | 01 |
| 238 | 54 | ) |
| 239 | 22 | INV |

PROGRAM 2 continued

```
240  23  LNX
241  95   =
242  42  STO
243  18   18
244  43  RCL
245  07   07
246  94  +/-
247  85   +
248  01   1
249  85   +
250  43  RCL
251  10   10
252  65   x
253  43  RCL
254  11   11
255  55   ÷
256  53   (
257  43  RCL
258  10   10
259  75   -
260  43  RCL
261  11   11
262  54   )
263  65   x
264  43  RCL
265  01   01
266  95   =
267  65   x
268  53   (
269  43  RCL
270  11   11
271  94  +/-
272  65   x
273  43  RCL
274  01   01
275  54   )
276  22  INV
277  23  LNX
278  95   =
279  44  SUM
280  18   18
281  71  SBR
282  47  CMS
283  91  R/S
284  76  LBL
285  23  LNX
286  43  RCL
287  11   11
288  45  YX
289  03   3
290  55   ÷
291  53   (
292  43  RCL
293  11   11
294  75   -
295  43  RCL
296  10   10
297  54   )
298  45  YX
299  03   3
300  95   =
301  42  STO
302  07   07
303  65   x
304  53   (
305  43  RCL
306  10   10
307  94  +/-
308  65   x
309  43  RCL
310  01   01
311  54   )
312  22  INV
313  23  LNX
314  85   +
315  53   (
316  43  RCL
317  07   07
318  94  +/-
319  85   +
320  01   1
321  85   +
322  53   (
323  43  RCL
324  11   11
325  75   -
326  43  RCL
327  11   11
328  45  YX
329  03   3
330  55   ÷
331  53   (
332  43  RCL
333  10   10
334  75   -
335  43  RCL
336  11   11
337  54   )
338  33  X²
339  54   )
340  65   x
341  43  RCL
342  01   01
343  85   +
344  43  RCL
345  10   10
346  65   x
347  43  RCL
348  11   11
349  33  X²
350  55   ÷
351  53   (
352  02   2
353  65   x
354  53   (
355  43  RCL
356  10   10
357  75   -
358  43  RCL
359  11   11
```

PROGRAM 2 continued

| | | |
|---|---|---|
| 360 | 54 | ) |
| 361 | 54 | ) |
| 362 | 65 | × |
| 363 | 43 | RCL |
| 364 | 01 | 01 |
| 365 | 33 | X² |
| 366 | 54 | ) |
| 367 | 65 | × |
| 368 | 53 | ( |
| 369 | 43 | RCL |
| 370 | 11 | 11 |
| 371 | 94 | +/- |
| 372 | 65 | × |
| 373 | 43 | RCL |
| 374 | 01 | 01 |
| 375 | 54 | ) |
| 376 | 22 | INV |
| 377 | 23 | LNX |
| 378 | 95 | = |
| 379 | 44 | SUM |
| 380 | 18 | 18 |
| 381 | 71 | SBR |
| 382 | 47 | CMS |
| 383 | 91 | R/S |
| 384 | 76 | LBL |
| 385 | 24 | CE |
| 386 | 43 | RCL |
| 387 | 12 | 12 |
| 388 | 65 | × |
| 389 | 33 | X² |
| 390 | 75 | - |
| 391 | 03 | 3 |
| 392 | 65 | × |
| 393 | 43 | RCL |
| 394 | 12 | 12 |
| 395 | 33 | X² |
| 396 | 65 | × |
| 397 | 43 | RCL |
| 398 | 10 | 10 |
| 399 | 95 | = |
| 400 | 55 | ÷ |
| 401 | 53 | ( |
| 402 | 53 | ( |
| 403 | 43 | RCL |
| 404 | 12 | 12 |
| 405 | 75 | - |
| 406 | 43 | RCL |
| 407 | 10 | 10 |
| 408 | 54 | ) |
| 409 | 65 | × |
| 410 | 33 | X² |
| 411 | 54 | ) |
| 412 | 95 | = |
| 413 | 42 | STO |
| 414 | 07 | 07 |
| 415 | 85 | + |
| 416 | 43 | RCL |
| 417 | 10 | 10 |
| 418 | 65 | × |
| 419 | 43 | RCL |
| 420 | 12 | 12 |
| 421 | 33 | X² |
| 422 | 55 | ÷ |
| 423 | 53 | ( |
| 424 | 43 | RCL |
| 425 | 12 | 12 |
| 426 | 75 | - |
| 427 | 43 | RCL |
| 428 | 10 | 10 |
| 429 | 54 | ) |
| 430 | 33 | X² |
| 431 | 65 | × |
| 432 | 43 | RCL |
| 433 | 01 | 01 |
| 434 | 95 | = |
| 435 | 65 | × |
| 436 | 53 | ( |
| 437 | 43 | RCL |
| 438 | 10 | 10 |
| 439 | 94 | +/- |
| 440 | 65 | × |
| 441 | 43 | RCL |
| 442 | 01 | 01 |
| 443 | 54 | ) |
| 444 | 22 | INV |
| 445 | 23 | LNX |
| 446 | 95 | = |
| 447 | 42 | STO |
| 448 | 18 | 18 |
| 449 | 53 | ( |
| 450 | 43 | RCL |
| 451 | 07 | 07 |
| 452 | 94 | +/- |
| 453 | 85 | + |
| 454 | 01 | 1 |
| 455 | 85 | + |
| 456 | 43 | RCL |
| 457 | 10 | 10 |
| 458 | 33 | X² |
| 459 | 65 | × |
| 460 | 43 | RCL |
| 461 | 12 | 12 |
| 462 | 55 | ÷ |
| 463 | 53 | ( |
| 464 | 43 | RCL |
| 465 | 10 | 10 |
| 466 | 75 | - |
| 467 | 43 | RCL |
| 468 | 12 | 12 |
| 469 | 54 | ) |
| 470 | 33 | X² |
| 471 | 65 | × |
| 472 | 43 | RCL |
| 473 | 01 | 01 |
| 474 | 54 | ) |
| 475 | 65 | × |
| 476 | 53 | ( |
| 477 | 43 | RCL |
| 478 | 12 | 12 |
| 479 | 94 | +/- |

PROGRAM 2 continued

| | | |
|---|---|---|
| 480 | 65 | × |
| 481 | 43 | RCL |
| 482 | 01 | 01 |
| 483 | 54 | ) |
| 484 | 22 | INV |
| 485 | 23 | LNX |
| 486 | 95 | = |
| 487 | 44 | SUM |
| 488 | 18 | 18 |
| 489 | 71 | SBR |
| 490 | 47 | CMS |
| 491 | 91 | R/S |
| 492 | 76 | LBL |
| 493 | 25 | CLR |
| 494 | 43 | RCL |
| 495 | 11 | 11 |
| 496 | 65 | × |
| 497 | 43 | RCL |
| 498 | 12 | 12 |
| 499 | 33 | X² |
| 500 | 95 | = |
| 501 | 55 | ÷ |
| 502 | 53 | ( |
| 503 | 43 | RCL |
| 504 | 11 | 11 |
| 505 | 75 | - |
| 506 | 43 | RCL |
| 507 | 10 | 10 |
| 508 | 54 | ) |
| 509 | 55 | ÷ |
| 510 | 53 | ( |
| 511 | 43 | RCL |
| 512 | 12 | 12 |
| 513 | 75 | - |
| 514 | 43 | RCL |
| 515 | 10 | 10 |
| 516 | 54 | ) |
| 517 | 33 | X² |
| 518 | 95 | = |
| 519 | 65 | × |
| 520 | 53 | ( |
| 521 | 43 | RCL |
| 522 | 10 | 10 |
| 523 | 94 | +/- |
| 524 | 65 | × |
| 525 | 43 | RCL |
| 526 | 01 | 01 |
| 527 | 54 | ) |
| 528 | 22 | INV |
| 529 | 23 | LNX |
| 530 | 95 | = |
| 531 | 42 | STO |
| 532 | 18 | 18 |
| 533 | 43 | RCL |
| 534 | 10 | 10 |
| 535 | 65 | × |
| 536 | 43 | RCL |
| 537 | 12 | 12 |
| 538 | 33 | X² |
| 539 | 55 | ÷ |
| 540 | 53 | ( |
| 541 | 43 | RCL |
| 542 | 10 | 10 |
| 543 | 75 | - |
| 544 | 43 | RCL |
| 545 | 11 | 11 |
| 546 | 54 | ) |
| 547 | 55 | ÷ |
| 548 | 53 | ( |
| 549 | 43 | RCL |
| 550 | 12 | 12 |
| 551 | 75 | - |
| 552 | 43 | RCL |
| 553 | 11 | 11 |
| 554 | 54 | ) |
| 555 | 33 | X² |
| 556 | 95 | = |
| 557 | 65 | × |
| 558 | 53 | ( |
| 559 | 43 | RCL |
| 560 | 11 | 11 |
| 561 | 94 | +/- |
| 562 | 65 | × |
| 563 | 43 | RCL |
| 564 | 01 | 01 |
| 565 | 54 | ) |
| 566 | 22 | INV |
| 567 | 23 | LNX |
| 568 | 95 | = |
| 569 | 44 | SUM |
| 570 | 18 | 18 |
| 571 | 43 | RCL |
| 572 | 10 | 10 |
| 573 | 65 | × |
| 574 | 43 | RCL |
| 575 | 11 | 11 |
| 576 | 55 | ÷ |
| 577 | 53 | ( |
| 578 | 43 | RCL |
| 579 | 10 | 10 |
| 580 | 75 | - |
| 581 | 43 | RCL |
| 582 | 12 | 12 |
| 583 | 54 | ) |
| 584 | 55 | ÷ |
| 585 | 53 | ( |
| 586 | 43 | RCL |
| 587 | 11 | 11 |
| 588 | 75 | - |
| 589 | 43 | RCL |
| 590 | 12 | 12 |
| 591 | 54 | ) |
| 592 | 95 | = |
| 593 | 42 | STO |
| 594 | 07 | 07 |
| 595 | 85 | + |
| 596 | 43 | RCL |
| 597 | 10 | 10 |
| 598 | 65 | × |
| 599 | 43 | RCL |

PROGRAM 2 continued

```
600  11   11
601  65   x
602  43  RCL
603  12   12
604  55   ÷
605  53   (
606  43  RCL
607  10   10
608  75   -
609  43  RCL
610  11   11
611  54   )
612  65   x
613  53   (
614  53   (
615  43  RCL
616  10   10
617  75   -
618  43  RCL
619  12   12
620  54   )
621  33  X²
622  35  1/X
623  75   -
624  53   (
625  43  RCL
626  11   11
627  75   -
628  43  RCL
629  12   12
630  54   )
631  33  X²
632  35  1/X
633  54   )
634  95   =
635  85   +
636  43  RCL
637  07   07
638  65   x
639  43  RCL
640  12   12
641  65   x
642  43  RCL
643  01   01
644  95   =
645  65   x
646  53   (
647  43  RCL
648  12   12
649  94  +/-
650  65   x
651  43  RCL
652  01   01
653  54   )
654  22  INV
655  23  LNX
656  95   =
657  44  SUM
658  18   18
659  71  SBR
660  47  CMS
661  91  R/S
662  76  LBL
663  10  E'
664  98  ADV
665  43  RCL
666  19   19
667  99  PRT
668  91  R/S
669  76  LBL
670  15   E
671  87  IFF
672  00   00
673  60  DEG
674  81  RST
675  76  LBL
676  60  DEG
677  22  INV
678  86  STF
679  00   00
680  86  STF
681  40  IND
682  00   00
683  00   0
684  42  STD
685  07   07
686  43  RCL
687  11   11
688  32  X:T
689  87  IFF
690  01   01
691  16  A'
692  87  IFF
693  02   02
694  48  EXC
695  87  IFF
696  03   03
697  49  PRD
698  87  IFF
699  04   04
700  50  IXI
701  76  LBL
702  48  EXC
703  43  RCL
704  10   10
705  67   EQ
706  16  A'
707  61  GTD
708  17  B'
709  76  LBL
710  49  PRD
711  43  RCL
712  10   10
713  67   EQ
714  16  A'
715  43  RCL
716  12   12
717  67   EQ
718  22  INV
719  61  GTD
```

```
720   17 B'
721   76 LBL
722   50 I×I
723   43 RCL
724   10  10
725   22 INV
726   86 STF
727   40 IND
728   00  00
729   22 INV
730   67  EQ
731   57 ENG
732   01   1
733   42 STO
734   07  07
735   76 LBL
736   57 ENG
737   43 RCL
738   12  12
739   22 INV
740   67  EQ
741   58 FIX
742   02   2
743   44 SUM
744   07  07
745   86 STF
746   40 IND
747   07  07
748   87 IFF
749   03  03
750   59 INT
751   22 INV
752   86 STF
753   40 IND
754   07  07
755   76 LBL
756   58 FIX
757   43 RCL
758   12  12
759   32 X:T
760   43 RCL
761   13  13
762   22 INV
763   67  EQ
764   59 INT
765   04   4
766   44 SUM
767   07  07
768   76 LBL
769   59 INT
770   86 STF
771   40 IND
772   07  07
773   87 IFF
774   00  00
775   17 B'
776   87 IFF
777   03  03
778   16 A'
779   87 IFF
780   04  04
781   25 CLR
782   87 IFF
783   05  05
784   24 CE
785   87 IFF
786   06  06
787   23 LNX
788   91 R/S
```

LABEL ADRESSES

```
005   18 C'
011   11  A
032   12  B
037   13  C
044   14  D
049   16 A'
067   28 LOG
095   29 CP
107   17 B'
113   39 COS
122   38 SIN
141   30 TAN
166   36 PGM
197   37 P/R
202   47 CMS
214   22 INV
285   23 LNX
385   24 CE
494   25 CLR
664   10 E'
671   15  E
677   60 DEG
700   48 EXC
708   49 PRD
720   50 I×I
734   57 ENG
754   58 FIX
767   59 INT
```

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